Summary: Calculating Derivatives

Derivatives of constant multiples

If $g(x) = k\ f(x)$ for some constant k, then

$$g'(x) = k\ f'(x)$$

at all points where f is differentiable.

Derivatives of sums

If $h(x) = f(x) + g(x)$, then

$$h'(x) = f'(x) + g'(x)$$

at all points where f and g are differentiable.

Derivatives of differences

Similarly, if $j(x) = f(x) - g(x)$, then

$$j'(x) = f'(x) - g'(x)$$

at all points where f and g are differentiable.

Derivatives of constant multiples - proof

Suppose that $g(x) = kf(x)$ for all x, where k is a constant. We want to prove that $g'(x) = kf'(x)$ at any point x where f is differentiable.

We know that

$$
\begin{aligned}
g'(x) &= \lim_{\Delta x \to 0} \frac{g(x + \Delta x) - g(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{kf(x + \Delta x) - kf(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} k\frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} k \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}.
\end{aligned}
$$

The first limit is just k, and the second limit is the definition of $f'(x)$. So we get $g'(x) = kf'(x)$.

What is linearity?

We've seen that differentiation "respects" addition and multiplication by a constant. That is, if you take a derivative of a sum of functions, you get the same thing as if you differentiated each part, and then added the derivatives. Similarly, if you take the derivative of k times a function, where k is a constant, then you get k times the derivative of the original function.

Respecting addition and constant multiplication in this way is called "linearity," and it is an important property of the derivative operation!

The Power Rule

If n is any fixed number, and $f(x) = x^n$, then $f'(x) = nx^{n-1}$.