Summary: Geometry of the derivative

Secant lines

The **secant line** of a function $f(x)$ over the interval $a \leq x \leq b$, is the line that passes through the points $(a, f(a))$ and $(b, f(b))$.

- The slope of the secant line is $\frac{f(b) - f(a)}{b - a}$, which is the average rate of change of the function $f(x)$ over the interval $a \leq x \leq b$.
- The equation for the secant line is $y = \frac{f(b) - f(a)}{b - a}(x - a) + f(a)$.

Tangent lines

The **tangent line** to a function $f(x)$ at the point $x = a$ is the line that passes through the point $(a, f(a))$, and whose slope is the instantaneous rate of change of $f(x)$ at the point $x = a$. This slope is the slope of the line you get if you imagine zooming in on the function until it looks like a line.

- The slope of the tangent line is $f'(a)$.
- The equation for the tangent line is $y = f'(a)(x - a) + f(a)$.

Properties of tangent lines

If the derivative of $f(x)$ exists at $x = a$, then the tangent line exists. The tangent line may exist if the derivative is undefined at $x = a$ though. (Example $f(x) = \sqrt[3]{x}$ has a vertical tangent line at $x = 0$.)

What a tangent line is, and is not

When introduced to tangent lines of circles, many students learn that a tangent is "a line that touches the curve in only one point." This is true if your curve is a circle, but for many other curves and functions, this is a **not a good** definition. See the examples below.