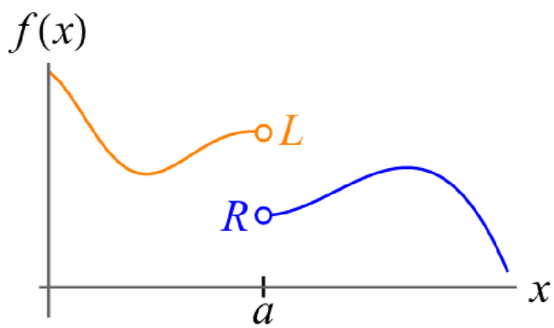
Summary: Introduction to limits

Definitions of right-hand and left-hand limits



Suppose $f(x)$ gets really close to R for values of x that get really close to (but are not equal to) a from the right. Then we say R is the **right-hand limit** of the function $f(x)$ as x approaches a from the right.

We write

$$f(x) \to R \text{ as } x \to a^+$$

or

$$\lim_{x\to \mathbf{a}^+} f(x) = R.$$

If $f(x)$ gets really close to L for values of x that get really close to (but are not equal to) a from the left, we say that L is the **left-hand limit** of the function $f(x)$ as x approaches a from the left.

We write

$$f(x) \to L \text{ as } x \to a^-$$

or

$$\lim_{x\to \mathbf{a}^-} f(x) = L.$$

Possible limit behaviors

There are many possible limit behaviors.

- The right-hand and left-hand limits may both exist and be equal.
- The right-hand and left-hand limits may both exist, but may fail to be equal.

- A right- and/or left-hand limit could fail to exist due to blowing up to $\pm\infty$. (Example: Consider the function $1/x$ near $x = 0$.) In this case, we either say the limit blows up to infinity. We also say that the limit does not exist because ∞ is not a real number!

- A right- and/or left-hand limit could fail to exist because it oscillates between many values and never settles down. In this case we say the limit does not exist.

Definition of the Limit

The Limit in Words If a function $f(x)$ approaches some value L as x approaches a from *both the right and the left*, then **the limit** of $f(x)$ exists and equals L.

The Limit in Symbols If

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L$$

then

$$\lim_{x \to a} f(x) = L.$$

Alternatively,

$$f(x) \to L \quad \text{as} \quad x \to a.$$

Remember that x is approaching a but does not equal a.

The Limit Laws:

Suppose $\lim_{x \to a} f(x) = L, \qquad \lim_{x \to a} g(x) = M.$

Then we get the following Limit Laws:

Limit Law for Addition:	$\lim_{x \to a} [f(x) + g(x)]$	$=$	$L + M$
Limit Law for Subtraction:	$\lim_{x \to a} [f(x) - g(x)]$	$=$	$L - M$
Limit Law for Multiplication:	$\lim_{x \to a} [f(x) \cdot g(x)]$	$=$	$L \cdot M.$

We also have part of the Limit Law for Division:

Limit Law for Division, Part 1: If $M \neq 0$, then $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M}$.

We will discuss what happens when $M = 0$ in a later section!